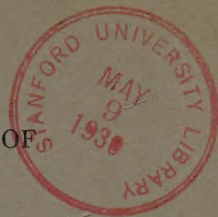


A METHOD OF OBTAINING THE OPTICAL CONSTANTS OF
METALLICALLY REFLECTING SUBSTANCES
IN THE INFRARED



DISSERTATION

SUBMITTED TO THE BOARD OF UNIVERSITY STUDIES OF THE JOHNS
HOPKINS UNIVERSITY IN CONFORMITY WITH THE REQUIREMENTS
FOR THE DEGREE OF DOCTOR OF PHILOSOPHY

BY

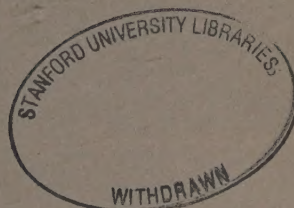
CARL BOECKNER

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A METHOD OF OBTAINING THE OPTICAL CONSTANTS OF METALLICALLY REFLECTING SUBSTANCES IN THE INFRARED

By C. BOECKNER

The method of obtaining the optical constants described in this paper consists in the measurement of the reflecting power of light (polarized in the plane of incidence) at two angles of incidence. A computation based on the Fresnel reflecting power formula then gives the optical constants; *i. e.*, the index of refraction and the extinction coefficient. The method is applicable to amorphous substances and to uniaxial crystals. In the latter case, however, only the "ordinary" reflecting power formula is derived. The method is applied to fused and crystalline quartz for wave lengths in their bands of metallic reflection at 8μ .

The usual katoptric method for obtaining these constants has not been applied to the infrared owing to the difficulty of constructing a polarizer and analyzer usable in this wave-length region. Nichols has succeeded in calculating the index of refraction of quartz for wave-lengths up to the reflection bands from measurements on absorption and normal reflecting power.¹ He could not penetrate the region of metallic reflection, however, owing to the impossibility of getting thin enough films.

Part of the object of the research was to investigate the marked change of reflecting power with change of angle of incidence observed by Gorton² and to see if this effect could be explained by the usual reflecting power formulas.

EXPERIMENTAL

The ratio of reflecting power of quartz to silver was measured, absolute reflecting power was not attempted.

Polarization of the light was brought about by means of a selenium mirror according to the method described by A. H. Pfund.³

The apparatus is shown diagrammatically in Fig. 1. A Nernst filament *A*, a concave mirror *B*, and the selenium mirror *C* are mounted on an arm of the spectrometer *D*. A supporting arrangement for the

¹ Nichols; Phys. Rev., 4, p. 297; 1896.

² A. F. Gorton, Phys. Rev., 7, p. 66; 1916.

³ A. H. Pfund, Phys. Rev., 22, 362; 1905.

quartz mirror *E* is mounted on the spectrometer table. The arm and the table can rotate independently, thus enabling the beam *EF* to the spectrometer *S* to be kept stationary for all angles of incidence of the light on the quartz.

The path of the light is as follows: Light from the filament *A* is rendered parallel by the mirror *B*. This parallel beam is polarized by *C* so that its direction of polarization is horizontal. The beam, perpendicular to the axis of rotation, falls on the vertical mirror *E*, is reflected to the fixed mirrors *F* and *G* and brought to focus on the slit *H* of the spectrometer *S*.

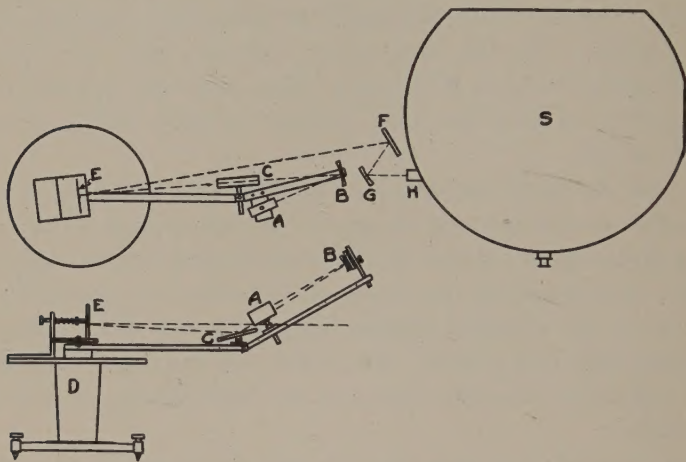


FIG. 1. *Arrangement for measuring reflecting power of polarized light for various angles of incidence in the infrared.*

The spectrometer is a rock salt instrument of moderate resolving power. The radiation was measured at the exit slit by means of a vacuum thermopile and a Leeds and Northrup galvanometer, sensitivity about 400 megohms. The angle of incidence was changed by rotating the spectrometer table through a known angle and rotating the arm *BAC* so that the filament image remained on the slit.

Measurements were made at 5° and 68° as angles of incidence. The silver mirror was prepared by chemical deposition of the silver on the glass, and its normal reflecting power taken from tables. The reflecting power at 68° was taken to be 99%. The mirrors were held against three fixed points in the usual way in order that the silver and the quartz surfaces could be accurately interchanged.

Stray light in the spectroscope was corrected for by obtaining the transmission of a very thin quartz bulb. The slight reading, about 1%, in the region of metallic reflection was taken to be the stray light. The effect of stray light was also diminished by the use of a glass shutter which is opaque to the 8 μ region.

A slit width correction described by Hyde⁴ was also applied. These corrections usually cancelled, amounting in the total to about 1%. They were not applied in all cases since they were of the order of magnitude of the experimental error.

**REFLECTING POWER—FUSED SiO_2 AT ANGLES OF INCIDENCE = 68° AND 50°
LIGHT POLARIZED $\parallel$ TO PLANE OF INCIDENCE**

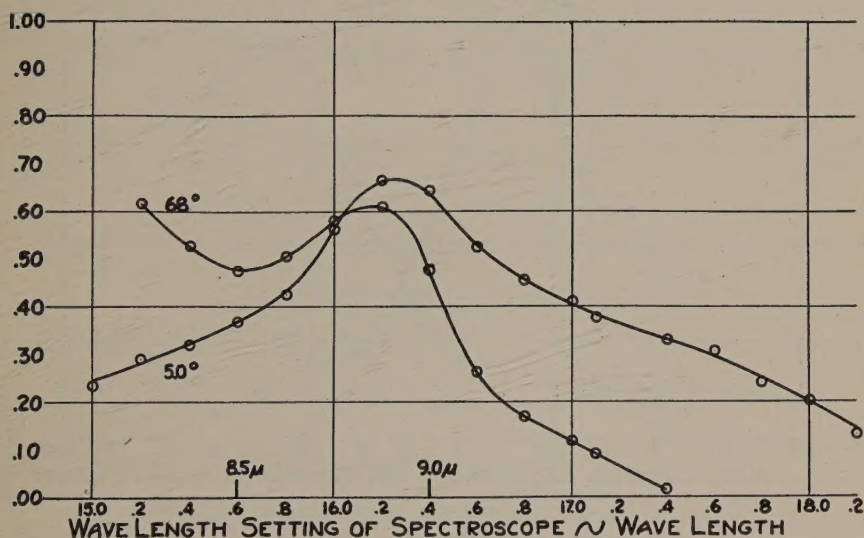


FIG. 2. Reflecting power fused SiO_2 light polarized $\parallel$ plane of incidence.

The maximum deflection obtained amounted to about 5 cms for 100% reflection on the galvanometer scale. This fact caused an uncertainty of about 1 or 2% in the results since readings could not be taken closer than $1/2$ mm.

The selenium mirror was adjusted to its polarizing angle by use of the fact that the vertical quartz mirror should completely extinguish the horizontally polarized light from the selenium at the polarizing angle of incidence of the quartz.

In the case of crystalline quartz a reflecting surface parallel to the

⁴ Hyde, *Astro Phys.*, 35, p. 237; 1912.

optic axis must be used and the plane of incidence of light must be perpendicular to this axis.

THE FRESNEL FORMULAS

The Fresnel formula for the reflection of light polarized in the plane of incidence is:

$$A = \tan(\theta - r) / \tan(\theta + r), \quad (1)$$

where

A is the amplitude for unit amplitude of incident light, θ the angle of incidence, r the angle of refraction and n the index of refraction.

FUSED SiO_2 - x = EXTINCTION COEFFICIENT
 v = INDEX OF REFRACTION

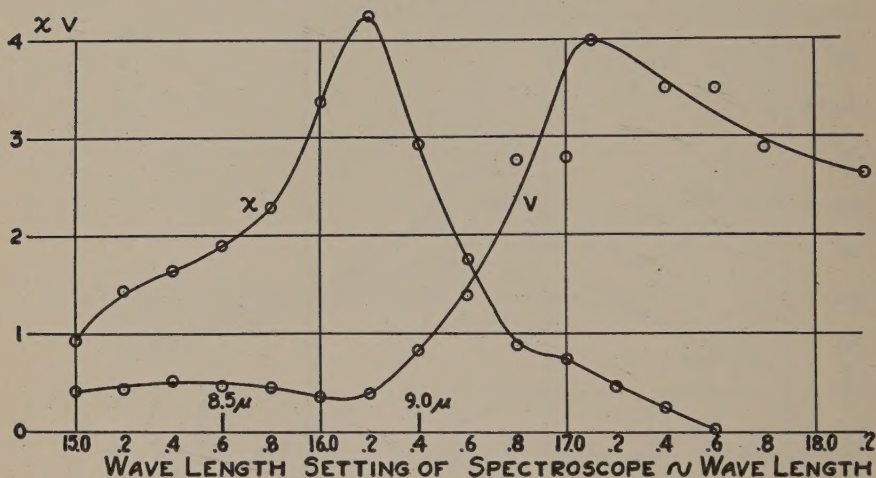


FIG. 3. Extinction coefficient and index of refraction fused SiO_2 .

For an absorbing medium n is complex, $n = v(1 - ix)$, where v is the index of refraction, and x is the extinction coefficient.

For the purpose of the experiment it is desired to find the absolute value of A since this is what is measured.

Quincke gives a first approximation to A valid for large values of vx^5

$$A^2 = (v^2(1+x^2) \cos^2 \theta - 2v \cos \theta + 1) / (v^2(1+x^2) \cos^2 \theta + 2v \cos \theta + 1) = R$$

where R is the fraction of the incident energy reflected.

From this relation the computation of v and x from measurements of the reflecting power at various angles of incidence is simple. If R_m

⁵ Winkelmann, Handbuch der Physik; 2 Aufl., p. 1300.

is the minimum reflecting power and θ_m the angle for minimum reflection, then

$$1+x^2=(1+R_m)/(1-R_m), \quad (2)$$

and

$$v=[1/\cos \theta_m][(1+R_m)/(1-R_m)]. \quad (3)$$

It was found, however, that the Quincke approximation did not represent the experimental results. This was found to be due to v being a small quantity, thus making vx in most cases less than one.

The exact expression for the absolute value of A in equation (1), however, was found to give results in agreement with the measurements. A comparatively simple expression for A can be found in the following way:

Let $1/n=(1/v)(1/v)/(1-ix)(1-ix)=Fe^{is}$, then

$$A=\tan(\theta-r)/\tan(\theta+r)=(\sin 2\theta-\sin 2r)/(\sin 2\theta+\sin 2r).$$

Let $r=a+ib$ then,

$$\begin{aligned} A &= \frac{\sin 2\theta - \sin 2a \cosh 2b - i \cos 2a \sinh 2b}{\sin 2\theta + \sin 2a \cosh 2b + i \cos 2a \sinh 2b}, \\ A^2 = R &= \frac{(\sin 2\theta - \sin 2a \cosh 2b)^2 + (\cos 2a \sinh 2b)^2}{(\sin 2\theta + \sin 2a \cosh 2b)^2 + (\cos 2a \sinh 2b)^2} \\ R &= \frac{\cosh^2 2b + \sin^2 2a - \cos^2 2\theta - 2 \sin 2\theta \sin 2a \cosh 2b}{\cosh^2 2b + \sin^2 2a - \cos^2 2\theta + 2 \sin 2\theta \sin 2a \cosh 2b}. \end{aligned} \quad (4)$$

To find a and b

$$\sin r = \sin(a+ib) = \sin \theta/n = F \sin \theta e^{is}$$

or

$$\sin a \cosh b + i \cos a \sinh b = F \sin \theta e^{is}.$$

Equating real and imaginary parts we have

$$\sin a \cosh b = F \sin \theta \cos s, \quad \cos a \sinh b = F \sin \theta \sin s.$$

On solving for $\sin a$, $\cosh b$

$$\sin a = (A-B)/2 \quad (5)$$

$$\cosh b = (A+B)/2 \quad (6)$$

Where

$$A^2 = 1 + F^2 \sin^2 \theta + 2F \sin \theta \cos s \quad (7)$$

and

$$B^2 = 1 + F^2 \sin^2 \theta - 2F \sin \theta \cos s. \quad (8)$$

It was found after the computations had been completed, that Drude

has given an exact form of the reflecting power formulas.⁶ The above form is somewhat more convenient for the purposes of the computations, however,

By examination of equations governing reflection from crystals it is found⁷ that Equation (1) applies to light reflected from a uniaxial crystal provided the light is polarized in the plane of incidence and provided the plane of incidence is perpendicular to the optic axis. The constants found will be those of the ordinary ray. Reflection at other surfaces involves hopeless complications in the formulas.

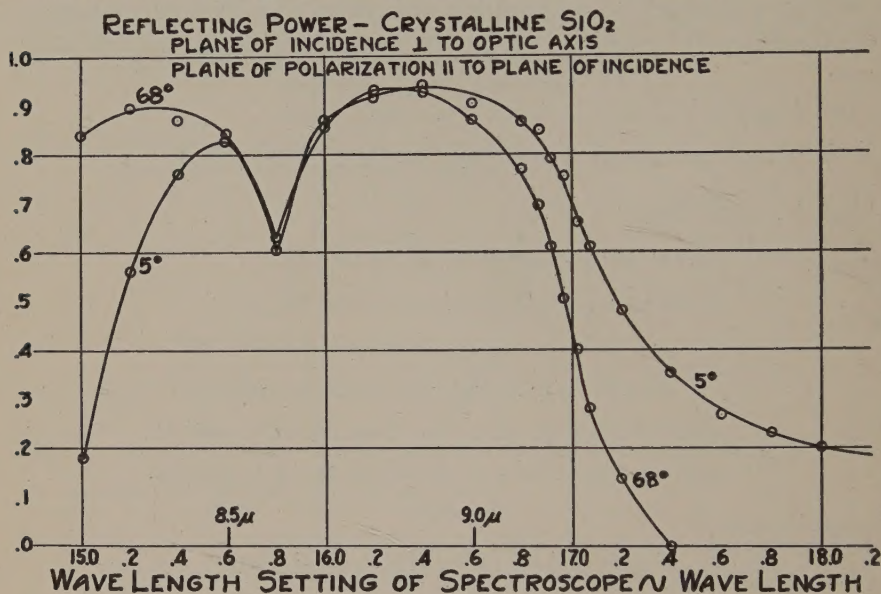


FIG. 4. Reflecting power crystalline SiO_2 plane of incidence perpendicular to optic axis light polarized perpendicular to plane of incidence.

Light polarized perpendicular to the plane of incidence can not be used in the case of an isotropic solid as its reflecting power varies too little with the angle of incidence to offer an accurate means of calculating v and x .

METHOD OF COMPUTATION

It, of course, is impossible to solve these equations explicitly for x and v . It was therefore necessary to resort to the method of trial and error. A value of F was assumed, $\cos s$ was then calculated from the reflection power at normal incidence by the relation

⁶ Ann. der Phys., u. Chem., 35, p. 520; 1888.

⁷ J. Walker; Analytical Theory of Light.

$$\cos s = [(1+F^2)/2F] [(1-R)/(1+R)].$$

A and B were then computed from (7-8) and $\sin a$ and $\cosh b$ obtained from equations (5-6). The reflection power at the oblique angle of incidence (68°) was then calculated from equation (4). A single such trial required about twenty minutes. It was found that two trials were usually sufficient to determine the F giving the observed reflecting power at the oblique angle of incidence. v and x are then obtained by the relations:

$$1+x^2=1/\cos^2 s \quad v=\cos s/F.$$

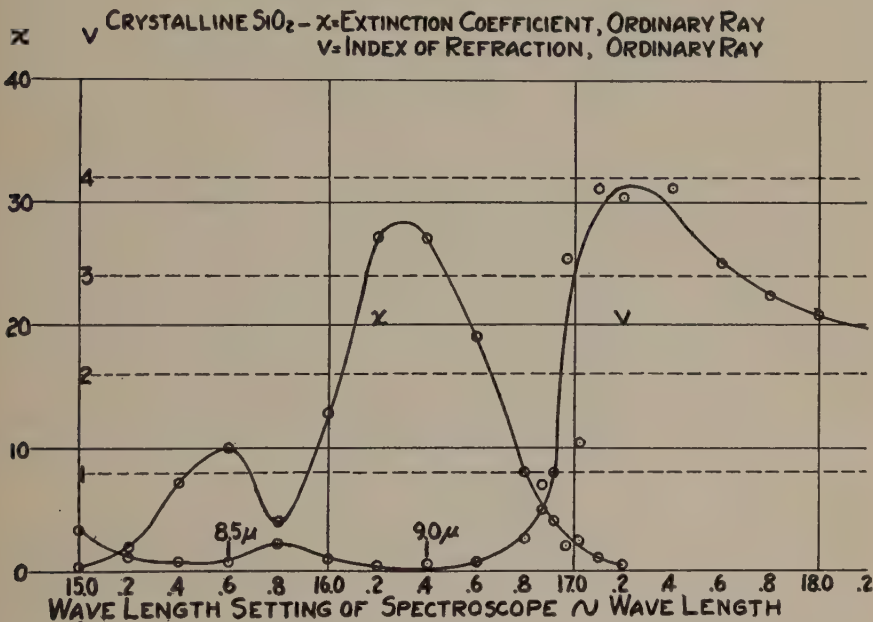


FIG. 5. Extinction coefficient and index of refraction crystalline SiO_2 .

ACCURACY OBTAINABLE

By examination of the Quincke approximation formula it was found that most accurate values of v and x are obtained by using measurements at normal incidence and at an angle as oblique as possible. An inherent disadvantage in the method is that an error in the calculated v and x is of the order of magnitude in the measured quantity $1+R/1-R$; if R is 95% a 1% error in measurement makes a 20% error in the calculated result. For fused silica, however, where the reflecting power does not go over 70%, a 1% error causes about a 4% error in the result.

RESULTS

The curves for the reflecting powers of fused and crystalline quartz are given for the two angles of incidence 5° and 68° . The wave lengths were obtained by assuming the reflecting maxima of crystalline quartz to occur at 9.0μ and 8.5μ .

TABLE 1. *Table showing agreement between calculated and observed reflecting power.*

Fused SiO_2		
Degrees	R obs.	R calc.
5	.57	.57
47.4	.51	.50
67.4	.58	.58
5	.32	.32
48	.34	.35
68	.47	.47
Crystalline SiO_2		
5	.74	.74
27.6	.72	.72
47.6	.80	.79
67.5	.89	.89

TABLE 2. *Table of extinction coefficients and indices of refraction.*

Crystalline SiO_2		
λ	x	v
8.13	.32	.40
8.25	1.8	.15
8.38	7.3	.12
8.50	10.	.15
8.63	4.0	.29
8.75	13.	.12
8.83	27.	.05
9.00	27.	.07
9.13	19.	.06
9.25	8.	.35
9.38	2.0	3.0
9.50		3.9
9.75		3.2
10.00		3.6

Fused SiO_2

λ	x	v
8.13	.94	.40
8.25	1.45	.44
8.38	1.65	.52
8.50	1.90	.49
8.63	2.3	.45
8.75	3.4	.36
8.83	4.3	.38
9.00	2.9	.83
9.13	1.8	1.40
9.25	.90	2.5
9.38	.75	3.7
9.50	.50	3.9
9.63	.25	3.5

Curves of the computed values of the index of refraction and extinction coefficient are given through the band of metallic reflection.

It is seen from the curves that the fused and the crystalline quartz have about the same maximal value for their index of refraction (about 4.) at about the same wave length. The maximum extinction coefficient is much greater for the crystalline quartz than for the fused quartz, being 27 for the former and 4 for the latter. Both specimens of quartz show two maxima in the index of refraction, the first absorption band for fused quartz is not resolved.

Data are given in Table 1 illustrating the degree to which the Fresnel metallic reflection formula represents the observed variation of reflecting power with the angle of incidence. The constants of the formula are calculated from the reflecting power at normal incidence (5°) and at (68°). The reflecting powers at some intermediate angles are computed and compared with the measured values. The agreement is within the error of measurement.

The author wishes to express his indebtedness to Professor A. H. Pfund for suggesting the problem and for help in the experimental part of the work.

THE JOHNS HOPKINS UNIVERSITY,
BALTIMORE, MD. APRIL, 1929.

VITA

Carl Boeckner, son of Edward and Caroline (Vierheller) Boeckner, was born in Baltimore, Maryland, May 11, 1903. He received his elementary and preparatory education in the public schools of Baltimore City and entered in the fall of 1921 the Johns Hopkins University. Here he pursued a Chemical Engineering course and was awarded the degree of Bachelor of Science in Chemistry in 1924.

In the fall of the same year he undertook graduate work in Physics and Mathematics, studying under Professor Ames, Professor Wood and Associate Professors Pfund and Murnaghan. He was awarded the degree of Master of Arts in 1926.

